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The cubic along the line a, b, c is $x^3-\lambda x-abc=0$, Differentiating this as to x, we have $3x^2-\lambda=0$, the roots of which are the polar pair of infinity, the intersection of the two lines D_1 and D_2. Calling the roots f and f', we have

$$f+f'=0, \text{ and } ff'=-f^2=-f'^2=-\tfrac{1}{3}\lambda. \quad \therefore f^2=f'^2=\tfrac{1}{3}\lambda.$$

By observation we see then that

$$\pi'_1\ a'=\pi'_3\ b'=\pi'_5\ c'=\tfrac{1}{3}\lambda;$$

so the new triad and the counter-triad of the original triad are in an involution whose double points are the polar pair of the intersection of the lines D_1 and D_2 as to the original triad.

From the results in § 9—11, we see that *in this form* by revolving the three lines on Σ, keeping the triad equispaced, we cut out, at each instant, along D_2 a triad of points having the same Hessian pair and being in the same involution whose double-points are the polar pair of the intersection of the Hessian diagonals as to the original triad.

Thus, we generate a pencil of point-triads along each of two lines, and dualistically of line-triads on each of two points.

THE TRISECTION PROBLEM.

By J. S. BROWN, Southwest Texas State Normal School, San Marcos, Texas.

The solution of this problem by means of the quadratrix, conchoid, and the cardioid are well known, and statements of the fact that the problem has been solved by means of the hyperbolic curve are equally well known, but the writer has never seen a solution by the last named method.

Ball, in his History of Mathematics, says that Viviani solved the problem by means of the *equilateral* hyperbola, and that Vieta determined that its solution depends upon the solution of a cubic equation.

I am not aware that the solution by means of the ceroid [so called from its resemblance to a pair of horns] has ever before been given.

I. Solution by Means of the Hyperbolic Curve.

If a series of circles be drawn through two points A and B, and if BP be one third of the arc BPA and H and H' points in the perpendicular bisector of AB, the locus of the point P, as the circle varies in size, is an hyperbola, since $PB=2PH$ constantly.

As the curve is central, its general equation is

$$a^2y^2-b^2x^2=-a^2b^2, \qquad (1).$$

Dividing the members of this equation by b^2 puts it in the form

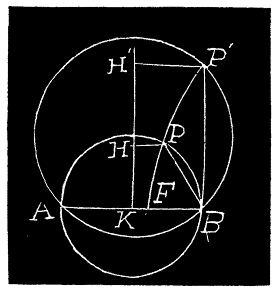
$$\left(\frac{a^2}{b^2}\right)y^2-x^2=-a^2, \qquad (2).$$

The factor $\frac{a^2}{b^2}=\cot^2\theta$, θ being the angle which the asymptote makes with the transverse axis. Therefore (2) becomes

$$y^2\cot^2\theta-x^2=-a^2, \qquad (3).$$

Since e (eccentricity)$=2$, $\cot^2\theta=\frac{1}{3}$ and θ is therefore 60°. Substituting this value of θ in (3) gives

$$\tfrac{1}{3}y^2-x^2=-a^2, \qquad (4).$$



Assuming AB as unity, the circle ABP' as one in which AB is the side of an inscribed square and K the mid point of AB as origin, $P'B=AB=1$, which is the ordinate of the point P', and the abscissa of this point is $a+\frac{1}{3}$.

Substituting these values of y and x in (4), gives

$$\tfrac{1}{3}-(a+\tfrac{1}{3})^2=-a^2, \qquad (5),$$

from which $a=\frac{1}{3}$. This value of a substituted in (4) gives $y^2-3x^2=-\frac{1}{3}$, the equation of the above described curve.

To trisect an angle by means of this curve, construct an isosceles triangle upon AB as a base, with the given angle as vertical angle.

The hyperbola will trisect the arc of the circle whose center is the vertex of this angle and whose radius is the leg of the triangle constructed.

II. Solution by Means of the Ceroid.

If a line be drawn through the center O of a circle to meet the circumference and also to meet a straight line LL', let us find the locus of the point P on HO, is HP is constantly equal to KP. Let O be the origin, d the distance from O to LL', $PH=PK=n$, and let $r=$the radius of the circle. We have

$$\frac{y}{d}=\frac{r+n}{r+2n}, \qquad (1),$$

from which is obtained the equation,

$$\frac{y^2}{d^2}(r+2n)^2=x^2+y^2, \qquad (2).$$

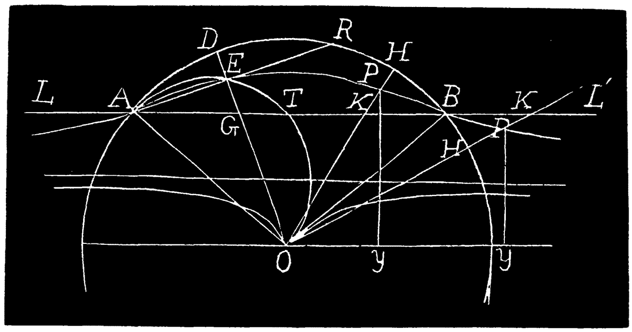
Also, $n=\sqrt{(x^2+y^2)}-r$, and substituting this value of n in (1) gives

$$\frac{y^2}{d^2}[2\sqrt{(x^2+y^2)}-r]^2=x^2+y^2, \qquad (3),$$

which is the equation of the required locus.

For convenience of discussion, (3) may be put in form

$$y^2[2\sqrt{(x^2+y^2)}-r-d][2\sqrt{(x^2+y^2)}-r+d]=d^2x^2, \qquad (4).$$



It is evident that the curve has two branches which meet at $\pm\infty$ on the asymptote whose equation is $y=\frac{1}{2}d$. n is regarded as negative when *in* the circle, and positive when *outside* of the circle.

The ceroid may be used to trisect any angle. For if AOB is a given angle, and ATO a semi-circle on AO as diameter, and E is the point common to the circumference ATO and the ceroid, the angle AOE is one third of the angle AOB.

Proof. $DE=GE$, for E is on the ceroid.

The angle AEO is a right angle. Hence angle DAE=angle GAE. Then arc AD=arc DR=arc RB. Therefore the angle AOD=one third of the angle AOB.

NOTE ON THE POSTULATE THAT A PART IS EQUIVALENT TO THE WHOLE.

By DR. G. A. MILLER.

The quadratic equations considered in elementary algebra are generally written in one of the following two forms:

$$ax^2+bx+c=0\ldots A,$$
$$ax^2+2bx+c=0\ldots B,$$